COURSE XXX

ATOMIC MOTION IN LASER LIGHT

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1. General Introduction

1.1. Purpose of this course

The purpose of this course is to discuss the basic processes and the physical mechanisms which govern atomic motion in laser light. During the last few years, spectacular results have been obtained concerning the possibility to "manipulate" atoms with laser light. A new expanding research field, called laser cooling and trapping, has come out (see for example the courses of W. Phillips, R. Blatt and H.Walther in this volume). In order to explore the limits of these new methods, several theoretical approaches have been developed. In this course, we review some of these approaches and we compare their advantages, their difficulties and their domains of validity.

The emphasis will be put here on physical ideas and physical mechanisms. The details of the calculations will not be given when they are avalaible in the litterature. We will just recall the principle of such calculations, devoting more time to the interpretation of the results and to the discussion of the various approximations which are introduced. We will consider only the case of neutral atoms. Laser cooling of ions is discussed in detail in the courses of R.Blatt and H.Walther.

1.2. The interacting systems

The atomic medium is supposed to be very dilute, so that one can ignore atom-atom interactions. We thus consider here a single atom A, with an excited state e and a ground state g separated by an energy interval

$$E_e - E_g = \hbar\omega_A \tag{1.1}$$

ω_A being called the atomic frequency. Important atomic observables are the electric dipole moment $\mathbf{d}$, the position $\mathbf{R}$ and the momentum $\mathbf{P}$ of the center of mass.This atom A is coupled, on the one hand, to the laser field L, and on the other hand to all the other modes of the radiation field which initially do not contain any photon and which form what we call the quantum vacuum field V (see Fig. 1).

Fig. 1. The interacting systems and their various couplings.

The laser field L is assumed to be monochromatic, with a frequency ω_L. If the initial state of the laser field is a coherent state, one can show (see Ref. 1 and exercise 17 in Ref. 2) that it is legitimate to describe it as a c-number external field

$$\mathbf{E}_L(\mathbf{r},t) = \epsilon(\mathbf{r})\mathcal{E}(\mathbf{r})\cos[\omega_L t + \Phi(\mathbf{r})] \tag{1.2}$$

where $\epsilon(\mathbf{r})$,$\mathcal{E}(\mathbf{r})$ and $\Phi(\mathbf{r})$ are respectively the polarization, the amplitude and the phase of the laser field in $\mathbf{r}$. The atom-laser coupling V_{AL} is characterized by the Rabi frequency Ω_1, which is proportional to the scalar product of the dipole moment matrix element $\langle e \mid \mathbf{d} \mid g \rangle$ by the laser field $\mathbf{E}_L(\mathbf{r},t)$. The hamiltonian evolution due to V_{AL} can be analyzed in terms of elementary processes of absorption and stimulated emission of laser photons by the atom.

The atom-vacuum field coupling V_{AV} is responsible for spontaneous emission of photons by the excited atom. It is characterized by the natural width Γ of the excited state e, which is also equal to the spontaneous emission rate of photons from e. Since V is a large system with an infinite number of degrees of freedom, the coupling V_{AV} introduces damping and fluctuations in the evolution of A. One of the main objectives of this course is to study the limits introduced by these fluctuations and to explain how it is possible to reduce them to their minimum value, and even to circumvent them.

Two extreme regimes can be considered for the evolution of A. For very short interaction times, i.e. for $t \ll \Gamma^{-1}$, one can neglect spontaneous emission, and the evolution of $A+L$ is described by a Schrödinger equation. For very long interaction times, i.e. for $t \gg \Gamma^{-1}$, several

spontaneous emission processes occur during the interaction time t, and the "reduced" evolution of A (traced over the vacuum field degrees of freedom) is then described by a master equation or by a Langevin equation. This second case is the most frequent one and it will be analyzed in detail in the following.

1.3. Characteristic times

For subsequent discussions, it will be useful to introduce here a few characteristic times and to compare their orders of magnitude.

The shortest time of the problem is the correlation time τ_c of the vacuum field. Vacuum fluctuations (see Ref. 3, Chap. 3 and Ref. 4, Chap. III) have a very broad frequency spectrum $J(\omega)$, which varies very slowly with ω around the atomic frequency ω_A : the typical frequency scale for the variations of $J(\omega)$ is ω_A itself. It follows that

$$\tau_c \simeq 1/\omega_A \tag{1.3}$$

The fact that τ_c is much shorter than all other characteristic times will allows us to consider the vacuum field V as a "reservoir" and to describe its effect on the evolution of the atom A as a relaxation process (see Ref. 5, Chap. 4 and references in and Ref. 2, Chap. IV).

For the atomic internal degrees of freedom, the most obvious characteristic time is the radiative lifetime τ_R of the excited state e

$$\tau_R = 1/\Gamma \tag{1.4}$$

which is the inverse of the natural width Γ of e and which can be considered as the relaxation time associated with spontaneous emission. The well known relation $\Gamma \ll \omega_A$ implies that $\tau_R \gg \tau_c$.

The existence of several Zeeman sublevels in the ground state gives rise to other internal relaxation times which are associated with optical pumping[6]. Absorption-spontaneous emission cycles, which are also called fluorescence cycles, can transfer the atom from one Zeeman sublevel g_m of g to another one $g_{m'}$. At low laser intensity I_L, it is possible to define a rate Γ' for the occurence of such optical pumping cycles , which is proportional to I_L. The inverse of this rate

$$\tau_P = 1/\Gamma' \tag{1.5}$$

is called the optical pumping time τ_P and can be considered as the mean time the atom has to wait before undergoing an optical pumping cycle. At low laser intensity I_L,

$$\tau_P \gg \tau_R \tag{1.6}$$

We will show in the second part of this course how the existence of such long internal relaxation times for multilevel atoms can give rise to very efficient new cooling mechanisms.

Note that, for two-level atoms, one can still define at low intensity a fluorescence rate Γ' and a mean time $1/\Gamma'$ between two fluorescence cycles experienced by the same atom, mean time which is much longer than τ_R. But such fluorescence cycles bring back the atom into the same ground state and they do not give rise to additional internal relaxation times. Actually, for a two-level atom, the only damping times appearing in the optical Bloch equations which describe the evolution of the internal degrees of freedom are all on the order of τ_R.

For the external (i.e. translational) atomic degrees of freedom, a very important characteristic time is the damping time of the atomic velocity. We will show in Chapter 4 (Section 4.1) that it is of the order of

$$T_{\text{ext}} = \hbar/E_R \tag{1.7}$$

where

$$E_R = \hbar^2 {k_L}^2/2M \tag{1.8}$$

is the recoil energy of the atom when it absorbs or emits a single laser photon. In (1.8), M is the total mass of the atom and $k_L = \omega_L/c$.

For most atomic transitions,

$$\hbar\Gamma \gg E_R \tag{1.9}$$

For example, for the resonance line of Sodium, $\hbar\Gamma = 400E_R$. When there is a single internal time $T_{\text{int}} = \tau_R$, it follows from (1.4), (1.7) and (1.9) that

$$T_{\text{ext}} \gg T_{\text{int}} \tag{1.10}$$

This separation of time scales introduces great simplifications in the analysis of atomic motion. As shown in the next chapters, one can then adiabatically eliminate the fast internal variables and derive reduced equations of motion for external variables.

However, it must be kept in mind that condition (1.10) in not always fulfilled. For atoms with degenerate ground states, the internal time τ_P can become, at low intensity, comparable to the external time (1.7), and even longer. External times shorter than (1.7) can also appear, such as the oscillation period T_{osc} of the atom in the bottom of an optical potential well. In such cases, it is no longer possible to eliminate internal variables, and the theoretical analysis is more complicated. But,

as shown in the last part of this course, such situations are also quite interesting, since they generally lead to much lower limits for the temperatures which can be achieved by laser cooling.

1.4. Outline of the course

In the first part of this course, i.e. from chapter 2 to chapter 7, we restrict ourselves to atoms with a nondegenerate ground state. This is for example the case for a transition $J_g = 0 \longleftrightarrow J_e = 1$. If we suppose in addition that a high static magnetic field $\mathbf{B}$ is applied, which pushes the two Zeeman sublevels $\mid e, m = \pm 1 \rangle$ very far away from resonance, we are left with a two-level atom $\{\mid e \rangle, \mid g \rangle\}$, with $\mid e \rangle = \mid e, m = 0 \rangle$ and $\mid g \rangle = \mid g, m = 0 \rangle$. The only nonzero matrix elements of the dipole moment operator $\mathbf{d}$ can then be written

$$\langle e \mid \mathbf{d} \mid g \rangle = d\, \epsilon_z = \langle g \mid \mathbf{d} \mid e \rangle \tag{1.11}$$

where we have assumed that d is real and where ϵ_z is the unit vector along the $0z$ axis. Note however that some papers[7] keep the three Zeeman sublevels $\mid e, m = -1, 0, +1 \rangle$ of the excited state with $\mathbf{B} = \mathbf{0}$.

Assuming that the atomic wave packet is very well localized in the laser wave (semiclassical limit), we first derive in Chapter 2 the expression of the radiative force operator which governs the motion of the center of the wave packet. The mean value of the force operator is then analyzed for an atom initially at rest (Chapter 3) and for a moving atom (Chapter 4), which allows us to introduce the notions of reactive, dissipative and friction forces. The fluctuations of the radiative force around its mean value are responsible for a diffusion of atomic momentum which heats the atom and which limits the efficiency of laser cooling. These fluctuations are studied both in the Heisenberg picture and in the Schrödinger picture (Chapter 5). All these results are now well known and we just present here a brief sketch of their derivation, referring the reader to existing publications for more details. On the other hand, we devote more time to the discussion of the physical mechanisms. In particular, we present in chapter 6 original results concerning the intriguing problem of an atom put at the node of a laser standing wave. We show that the anomalously large momentum diffusion which occurs in such a place (where there is no light) is due to interference effects between different scattering amplitudes and reveals the existence of a new kind of "*correlated redistribution*" process. The physical mechanisms occurring at high intensity are also analyzed in Chapter 7, using the so called dressed atom approach.

The second part of the course (Chapters 8 to 11) deals with atoms having several Zeeman sublevels in the ground state. We consider for example simple atomic transitions with $J_g = 1/2$ or $J_g = 1$. We first recall in Chapter 8 a few basic results concerning the effect of a weak intensity light irradiation on the internal dynamics of a slowly moving multilevel atom. Several effects, such as optical pumping and light-shifts are briefly reviewed. We then show how these effects can conspire to improve the efficiency of laser cooling by orders of magnitude. Two recent developments are studied in detail. The first one concerns laser cooling with laser configurations exhibiting strong polarization gradients (Chapter 9 and 10). The physical mechanisms responsible for the very low temperatures which have been recently measured (a few microkelvins) are analyzed. Some new results concerning the limits of polarization gradient cooling are presented. Finally, we discuss in Chapter 11 a method using velocity selective coherent population trapping for cooling atoms below the so called recoil limit.

PART I : TWO-LEVEL ATOMS

2. Radiative force in the semiclassical limit

We will follow in this chapter the presentation of Ref. 8.

2.1. Hamiltonian

The hamiltonian H of the global system represented on Fig. 1 can be written

$$H = H_A + H_V + V_{AL} + V_{AV} \tag{2.1}$$

The first term

$$H_A = H_A^{\text{ext}} + H_A^{\text{int}} = \frac{\mathbf{P}^2}{2M} + \hbar\omega_A \mid e\rangle\langle e \mid \tag{2.2}$$

is the atomic hamiltonian, which is the sum of the kinetic energy of the center of mass and of the internal energy. The second term

$$H_V = \sum_j \hbar\omega_j (a_j^+ a_j + \tfrac{1}{2}) \tag{2.3}$$

is the energy of the quantum radiation field (see Ref. 4, Chap. III), initially in the vacuum state, expressed as a sum of contributions of the various modes j. Note that , since the laser field L is treated here as a c-number external field, there is no hamiltonian H_L for L in (2.1). The third term of (2.1) is the coupling

$$V_{AL} = -\mathbf{d} \cdot \mathbf{E}_L(\mathbf{R}, t) \tag{2.4}$$

between the atomic dipole moment $\mathbf{d}$ and the laser electic field $\mathbf{E}_L(\mathbf{R}, t)$ given in (1.2) and evaluated at the position $\mathbf{R}$ of the center of mass (electric dipole approximation). Finally, the last term of (2.1) describes the atom-vacuum field coupling

$$V_{AV} = -\mathbf{d} \cdot \mathbf{E}(\mathbf{R}) \tag{2.5}$$

where the mode expansion of the electric field operator (see Ref. 4, Chap. III) is given by

$$\mathbf{E}(\mathbf{r}) = i \sum_j \mathcal{E}_j a_j \epsilon_j e^{i\mathbf{k}_j \cdot \mathbf{r}} + h.c. \tag{2.6}$$

a_j^+ (and a_j) being the creation (and annihilation) operators of a photon of momentum $\hbar\mathbf{k}_j$, energy $\hbar\omega_j = \hbar c k_j$ and polarization ϵ_j, and $\mathcal{E}_j$ being a normalization constant equal to

$$\mathcal{E}_j = \sqrt{\frac{\hbar\omega_j}{2\epsilon_0 L^3}} \tag{2.7}$$

(L^3 is the quantization volume).

One very often uses the so called "rotating wave approximation" (r.w.a.) which consists of neglecting the "antiresonant" terms of V_{AL} and V_{AV}. Using for $\mathbf{d}$ the expression

$$\mathbf{d} = d\ \epsilon_z\ (\ |\ e\rangle\langle\ g\ |\ +(|\ g\rangle\langle\ e\ |\ ) \tag{2.8}$$

which results from (1.11), introducing the *Rabi frequency* Ω_1 given by

$$\hbar\Omega_1(\mathbf{r}) = -d\ \mathcal{E}(\mathbf{r})\ \epsilon_z.\epsilon(\mathbf{r}) \tag{2.9}$$

and neglecting the antiresonant terms $e^{-i\omega_L t}\ |\ g\rangle\langle\ e\ |$ (and *h.c.*) of V_{AL} then leads to

$$V_{AL} = \frac{\hbar\Omega_1(\mathbf{R})}{2}\left[\ e^{-i\Phi(\mathbf{R})}\ e^{-i\omega_L t}\ |\ e\rangle\langle\ g\ |\ +\ h.c.\ \right] \tag{2.10}$$

One can similarly neglect the antiresonant terms $a_j\ |\ g\rangle\langle\ e\ |$ (and *h.c.*) of V_{AV}.

2.2. Heisenberg equations

In order to study the dynamics of the center of mass of A, we start from the Heinsenberg equations for $\mathbf{R}$ and $\mathbf{P}$. The equation of motion of $\mathbf{R}$ is

$$\dot{\mathbf{R}} = \frac{1}{i\hbar}[\,\mathbf{R}, H\,] = \frac{\partial H}{\partial \mathbf{P}} = \frac{\mathbf{P}}{M} \tag{2.11.a}$$

and shows that $\mathbf{P}/M$ is the velocity of the center of mass. It follows that the force operator, $\mathbf{F}(\mathbf{R}) = M\ddot{\mathbf{R}} = \dot{\mathbf{P}}$, is given by the Heisenberg equation for $\mathbf{P}$

$$\begin{aligned}\dot{\mathbf{P}} = M\ddot{\mathbf{R}} &= \frac{1}{i\hbar}[\,\mathbf{P}, H\,] = -\frac{\partial H}{\partial \mathbf{R}} \\ &= -\nabla V_{AL}(\mathbf{R}) - \nabla V_{AV}(\mathbf{R}) \\ &= \text{Force operator } \mathbf{F}(\mathbf{R})\end{aligned} \tag{2.11.b}$$

The quantum electric field operator $\mathbf{E}(\mathbf{R})$ appearing in $\nabla V_{AV}(\mathbf{R})$ can be transformed using the Heisenberg equation for $a_j(t)$. The general solution of this equation, which is a linear differential equation with a source term, can be written (see Ref. 9 and Ref. 2, Complement A_V)

$$a_j(t) = a_j(0)\ e^{-i\omega_j t} + a_j^{\text{source}}(t) \tag{2.12}$$

where the first term

$$a_j^{\text{vac}}(t) = a_j(0)\ e^{-i\omega_j t} \tag{2.13}$$

is the general solution of the homogeneous equation and corresponds to the vacuum field evolving freely between the initial time $t = 0$ and t, and where the second term is a particular solution of the inhomogeneous equation which corresponds to the "source field" originating from the atomic dipole moment between $t = 0$ and t. Inserting (2.12) into the mode expansion (2.6) of $\mathbf{E}(\mathbf{R}, t)$ allows one to separate two contributions in the electric field operator

$$\mathbf{E}(\mathbf{R}, t) = \mathbf{E}^{\text{vac}}(\mathbf{R}, t) + \mathbf{E}^{\text{source}}(\mathbf{R}, t) \tag{2.14}$$

corresponding respectively to the vacuum free field and to the source field.

In all previous expressions, the *total* field operator $a_j(t)$ commutes with all atomic operators taken at the same time, since field and atomic operators commute at $t = 0$ (they act in different spaces), and since the unitary hamiltonian evolution between $t = 0$ and t preserves the commutation relations. All possible orders between $a_j(t)$ and atomic operators are thus equivalent. This is no longer true for $a_j^{\text{vac}}(t)$ and $a_j^{\text{source}}(t)$ separately. Depending on the choice made initially for ordering the $a_j(t)$ and the atomic operators, the respective contributions of the vacuum field and of the source field will appear to be different, wheras their *sum* of course does not depend on this initial choice (see Ref. 9 and Ref. 2, Complement A_V). From now on, we will choose the *normal order*, where all the annihilation operators $a_j(t)$ are put at the extreme right, and all the creation operators $a_j^\dagger(t)$ at the extreme left. Such an order leads in general to simpler calculations, in particular when one takes average values in the vacuum state $| 0 \rangle$ of the quantum field. As a consequence of the well known relations

$$a_j(0)\ |\ 0\ \rangle = 0 \qquad\qquad \langle\ 0\ |\ a_j^\dagger(0) = 0 \tag{2.15}$$

the contribution of the vacuum field to the vacuum average values vanishes. It must be kept in mind however that other orders may be useful. For example, the completely symmetrical order is more convenient for physical interpretations[10].

We insert now (2.14) into the second term, $-\nabla V_{AV}(\mathbf{R})$, of the second line of (2.11.b). One can show that the source field $\mathbf{E}^{\text{source}}(\mathbf{r},t)$ due to the atomic dipole moment $\mathbf{d}$ has no gradient at the position $\mathbf{R}$ where this dipole moment is located (this field is an even function of $\mathbf{r}-\mathbf{R}$). The contribution of the source field to the force operator thus vanishes, and we get finally

$$\mathbf{F}(\mathbf{R},t) = -\nabla V_{AL}(\mathbf{R},t) - : \nabla V_{AV}^{\text{vac}}(\mathbf{R},t) : \tag{2.16}$$

where V_{AV}^{vac} is obtained from V_{AV} by replacing the total field by the vacuum field and where the notation $: X :$ means that the normal order has been chosen for ordering X.

2.3. Semiclassical limit

Up to now, no assumption has been made concerning the atomic wave packet. We now assume, as in References 8 and 11, that such a wave packet is sufficiently well localized in position space and in momentum space to allow the quantum description of atomic motion to be as close as possible to the classical description where the atom has a well defined position and a well defined momentum.

2.3.1. Conditions for having a localized atomic wave packet at a given time

At $t = 0$, the external atomic state is supposed to be described by a wave function $\psi(\mathbf{r})$ centered on

$$\mathbf{r}_0 = \langle \mathbf{R}(0) \rangle \tag{2.17}$$

and having a width $\Delta R(0)$. In momentum space, the same state is described by a wave fonction centered on

$$\mathbf{p}_0 = \langle \mathbf{P}(0) \rangle \tag{2.18}$$

with a width $\Delta P(0)$ related to $\Delta R(0)$ by the Heisenberg inequality

$$\Delta R(0)\, \Delta P(0) \geq \hbar \tag{2.19}$$

We will say a few words below (in Subsection 2.3.2) on the more general case where the external state is a stastistical mixture described by a density operator rather than a pure state described by a wave function.

The force exerted by the laser wave on the atom varies over distances on the order of the laser wavelength λ_L, or larger. It also depends on the velocity v of the atom because of the Doppler effect $k_L v$, where $k_L = 2\pi/\lambda_L$. The velocity change δv producing an appreciable change of the atomic response to the laser excitation is such that $k_L \delta v$ is on the order of the natural width Γ of the excited state, or larger.

If one wants the force experienced by the atomic wave packet to be quasiclassisal, i.e. with very small fluctuations around its mean value, two conditions must be fulfilled. First, the position spread $\Delta R(0)$ must be small compared to λ_L

$$\Delta R(0) \ll \lambda_L \qquad \text{or equivalently} \qquad k_L \Delta R(0) \ll 1 \tag{2.20}$$

Secondly, the velocity spread $\Delta v(0) = \Delta P(0)/M$ must be small enough to allow the corresponding spread of Doppler shifts to be negligible compared to Γ.

$$\frac{k_L \Delta P(0)}{M} \ll \Gamma \tag{2.21}$$

Note that condition (2.21) does not imply any relation between Γ and the mean Doppler effect $k_L p_0/M$ of the wave packet. Such a mean Doppler effect may be large compared to Γ. Condition (2.21) bears on the spread of Doppler shifts, not on the mean Doppler shift.

Equations (2.20) and (2.21), which express the localization of the wave packet in position space and in momentum space, impose upper bounds on $\Delta R(0)$ and $\Delta P(0)$, which can be in conflict with the Heisenberg inequality (2.19). Multiplying both sides of (2.20) by the corresponding both sides of (2.21) and using (2.19), we get the compatibility condition

$$\frac{\hbar k_L^2}{M} \ll \Gamma \tag{2.22}$$

One finds again the condition $E_R \ll \hbar\Gamma$, written in (1.9), and equivalent to $T_{\text{ext}} \gg T_{\text{int}}$ (see Equ. (1.10)). The existence of two time scales thus appears as a necessary condition for the semiclassical limit.

2.3.2. Is localization maintained at later times ?

Conditions (2.20) and (2.21) have been imposed at time $t = 0$. Can we still consider the atom as well localized both in position and momentum at a later time τ?

Suppose first that $\tau \ll T_{\text{ext}}$, so that one can neglect the change of atomic momentum between $t = 0$ and $t = \tau$ (remember that T_{ext} is the damping time of **P**). One can thus write

$$P(\tau) \simeq P(0) \tag{2.23.a}$$

from which one deduces, using (2.11.a)

$$R(\tau) \simeq R(0) + \frac{P(0)}{M}\tau \tag{2.23.b}$$

It follows that

$$\Delta P(\tau) \simeq \Delta P(0) \tag{2.24.a}$$

$$\Delta R(\tau) \simeq \Delta R(0) + \frac{\Delta P(0)}{M}\tau \tag{2.24.b}$$

Because of (2.24.a), momentum localization is unchanged. Equ. (2.24.b) describes the well known spatial spreading of the wave packet. In order to maintain spatial localization at time $t = \tau$, one must have

$$\frac{k_L \Delta P(0)}{M}\tau \ll 1 \tag{2.25}$$

which means that τ must not be too long. If $\tau \simeq T_{\text{int}} \simeq \Gamma^{-1}$, one easily checks that (2.25) is equivalent to (2.21). The spatial spreading of the atomic wave packet during a time on the order of Γ^{-1}, or a few Γ^{-1}, is thus negligible. It follows that one can choose time intervals τ such that

$$T_{\text{int}} \ll \tau \ll T_{\text{ext}} \tag{2.26}$$

which are short enough compared to T_{ext} so that one can neglect the variations of atomic momentum during τ, and sufficiently long compared to T_{int} to allow the internal degrees of freedom to reach an equilibrium state. It is therefore possible to use the concept of steady-state force for a well localized wave packet.

At much longer times, $\tau \gg T_{\text{ext}}$, it is no longer possible to consider that the atomic momentum has not changed. Because of the random

character of the momentum exchanges between the atom and the field, there is a momentum diffusion which tends to increase ΔP. But if the laser frequency is properly tuned, there is also a laser cooling which tends to reduce ΔP. One can show (see Subsection 5.2.4) that, as a result of the competition between these two processes, ΔP tends to values which still satisfy (2.21). There are therefore situations where the atomic momentum remains well localized. By contrast, and except for a few special cases where the atom is strongly confined by a trapping potential (for example, near the nodes of an intense standing wave), it seems impossible to maintain equation (2.20) for all times. Spatial diffusion tends in general to increase ΔR well above λ_L.

There is however an important point which is overlooked by such an analysis. At long times, the state of the center of mass can certainly no longer be described by a wave function, even if this was true at $t = 0$. Several fluorescence cycles have occurred and quantum nonseparable correlations have appeared between the various degrees of freedom, with the result that the reduced state of the center of mass is a statistical mixture of states described by a density operator σ_A^{ext}, rather than a pure state described by a wave function. In such a case, the characterization of spatial localization by condition (2.20) is too crude, and a more precise definition must be given. Let $\langle \mathbf{r}' \mid \sigma_A^{\text{ext}} \mid \mathbf{r}'' \rangle$ be the density matrix representing σ_A^{ext} in the basis of the eigenstates of the position operator $\mathbf{R}$ of the center of mass. The width of the spatial distribution $\mathcal{R}(\mathbf{r}) = \langle \mathbf{r} \mid \sigma_A^{\text{ext}} \mid \mathbf{r} \rangle$, given by the diagonal elements of σ_A^{ext}, is the width ΔR considered above, which can increase well above λ_L. But there is also another important characteristic length, called the spatial coherence length ξ_A, and defined as the typical distance beyond which the off diagonal elements of σ_A^{ext} (spatial atomic coherences) vanish

$$\langle \mathbf{r}' \mid \sigma_A^{\text{ext}} \mid \mathbf{r}'' \rangle \simeq 0 \qquad \text{if} \qquad \mid \mathbf{r}' - \mathbf{r}'' \mid \gg \xi_A \tag{2.27}$$

Now, one can show that, in the problem considered here, ξ_A remains always much smaller than λ_L. This is due to the fact that scattering destroys spatial coherences[12]. Consider a target particle $\mathcal{T}$ that scatters at random times projectile particles $\mathcal{P}$ having a de Broglie wavelength λ_P. One can show[12] that the spatial coherence length of $\mathcal{T}$ is reduced to values much shorter than λ_P by these scattering processes. Here, the target is the atom A, the projectiles are the laser photons with wavelength λ_L, and the scattering processes correspond to fluorescence cycles occurring at random times. It follows that

$$\xi_A \ll \lambda_L \tag{2.28}$$

In order to understand the implications of (2.28), imagine that we express σ_A^{ext} at time $t = \tau$ as a statistical mixture of wave packets. Each of these individual wave packets must have a spatial extension much smaller than λ_L since, otherwise, (2.28) would be violated; and the centers of these wave packets are distributed over an interval ΔR, which is the width of the spatial distribution and which can be much larger than λ_L. Such an analysis shows that we can consider that localization is maintained for all times, but this localization concerns the individual wave packets in terms of which the stastistical mixture can be expressed, and not the whole spatial distribution. The fact that we are obliged to consider several wave packets at time $t = \tau$, even if we start from a single wave packet at $t = 0$, is due to the randomness of fluorescence cycles, which introduces fluctuations in the atomic evolution. Actually, the various wave packets into which σ_A^{ext} can be decomposed at time $t = \tau$, can be considered as a stastistical ensemble, in the classical sense, representing the various possible "histories" which can happen to the atom between $t = 0$ and $t = \tau$.

2.4. Mean force and Langevin force

Suppose that, at $t = 0$, the atomic wave packet is well localized around $\mathbf{r}_0$, with a sufficiently small velocity spread around

$$\mathbf{v}_0 = \frac{\mathbf{p}_0}{M} \tag{2.29}$$

If we are interested in the rate of variation of the mean value of $\mathbf{P}$, $\mathrm{d}\langle\mathbf{P}\rangle/\mathrm{d}t$, and of the variance of $\mathbf{P}$, $\mathrm{d}(\Delta P)^2/\mathrm{d}t$, in the neighbourhood of $t = 0$, we must calculate one-time average values such as $\langle\mathbf{F}(\mathbf{R}(\tau),\tau)\rangle$ and two-time average values such as $\langle\mathbf{F}(\mathbf{R}(\tau),\tau)\cdot\mathbf{F}(\mathbf{R}(\tau'),\tau')\rangle$ where $\mathbf{F}$ is the force operator defined in (2.16) and where τ and τ' are times close enough to 0, i.e. much smaller than T_{ext} (but which can eventually be as large as a few $T_{\text{int}} = \Gamma^{-1}$). Since $\tau, \tau' \ll T_{\text{ext}}$, we can use (2.23.b) for reexpressing $\mathbf{R}(\tau)$ and $\mathbf{R}(\tau')$ as a function of $\mathbf{R}(0)$, $\mathbf{P}(0)$, τ, τ'. Furthermore, since the wave packet is well localized in position and in momentum, we can replace in the one-time and two-time averages the operators $\mathbf{R}(0)$ and $\mathbf{P}(0)$ by the c-numbers $\mathbf{r}_0$ and $\mathbf{p}_0$. It thus appears that, for calculating $\mathrm{d}\langle\mathbf{P}\rangle/\mathrm{d}t$ and $\mathrm{d}(\Delta P)^2/\mathrm{d}t$ around $t = 0$, we can replace $\mathbf{F}(\mathbf{R}(\tau),\tau)$ by $\mathbf{F}(\mathbf{r}_0+\mathbf{v}_0\tau,\tau)$. Note that $\mathbf{F}(\mathbf{r}_0+\mathbf{v}_0\tau,\tau)$ acts only on internal atomic variables and field variables whereas $\mathbf{F}(\mathbf{R}(\tau),\tau)$ acts also on external variables.

The rate of variation of $\langle \mathbf{P} \rangle$ is equal to the mean value of $\mathbf{F}(\mathbf{r}_0+\mathbf{v}_0 t, t)$, which we note $\mathcal{F}(\mathbf{r}_0+\mathbf{v}_0 t, t)$. Since the mean value of the second term of the right-hand side of (2.16) vanishes because of the normal order (see (2.15)), we get for the mean force

$$\mathcal{F}(\mathbf{r}_0+\mathbf{v}_0 t, t) = -\langle \nabla V_{AL}(\mathbf{r}, t) \rangle|_{\mathbf{r}=\mathbf{r}_0+\mathbf{v}_0 t} \tag{2.30}$$

The fluctuating part of $\mathcal{F}(\mathbf{r}_0+\mathbf{v}_0 t, t)$, i.e. the difference between the force and its mean value,

$$\delta\mathbf{F}(\mathbf{r}, t) = \mathbf{F}(\mathbf{r}, t) - \mathcal{F}(\mathbf{r}, t) \tag{2.31}$$

plays an important role in the calculation of $\mathrm{d}(\Delta P)^2/\mathrm{d}t$. Such a fluctuating force, with zero mean value, is called the Langevin force. Using (2.16) and (2.30), we get

$$\delta\mathbf{F}(\mathbf{r}, t) = \delta\mathbf{F}_{\mathrm{las}}(\mathbf{r}, t) + \delta\mathbf{F}_{\mathrm{vac}}(\mathbf{r}, t) \tag{2.32}$$

where

$$\delta\mathbf{F}_{\mathrm{las}}(\mathbf{r}, t) = -\nabla V_{AL}(\mathbf{r}, t) - \mathcal{F}(\mathbf{r}, t) \tag{2.33.a}$$

$$\delta\mathbf{F}_{\mathrm{vac}}(\mathbf{r}, t) = - : \nabla V_{AV}(\mathbf{r}, t) : \tag{2.33.b}$$

represent respectively the contributions of the laser field and of the vacuum field to the Langevin force.

We finally reexpress $-\nabla V_{AL}(\mathbf{r}, t)$. Using (2.10), we get

$$-\nabla V_{AL}(\mathbf{r}, t) = -\frac{\hbar}{2} \mid e \rangle\langle g \mid e^{-i\omega_L t} \nabla[\Omega_1(\mathbf{r}) e^{-i\Phi(\mathbf{r})}] + \ h.c. \tag{2.34}$$

The calculation of the gradient gives

$$\nabla[\Omega_1(\mathbf{r}) e^{-i\Phi(\mathbf{r})}] = \Omega_1(\mathbf{r}) e^{-i\Phi(\mathbf{r})} [\alpha(\mathbf{r}) - i\beta(\mathbf{r})] \tag{2.35}$$

where

$$\alpha(\mathbf{r}) = \frac{\nabla\Omega_1(\mathbf{r})}{\Omega_1(\mathbf{r})} \tag{2.36.a}$$

$$\beta(\mathbf{r}) = \nabla\Phi(\mathbf{r}) \tag{2.36.b}$$

characterize respectively the spatial variations of the Rabi frequency and of the phase. If we insert (2.35) and (2.36) in (2.34) and if we use (2.30)

and the fact that the mean value of $\mid e\rangle\langle g \mid$ is $\sigma_{ge}(t)$ where σ is the internal atomic density operator, we finally get the following expression for the mean force

$$\begin{aligned}\mathcal{F}(\mathbf{r},t) &= \mathrm{Re}\left\{\sigma_{ge}(t)\,\hbar\Omega_1(\mathbf{r})\,e^{-i[\omega_L t+\Phi(\mathbf{r})]}[\,\alpha(\mathbf{r})-i\beta(\mathbf{r})\,]\right\} \\ &= -\hbar\Omega_1(\mathbf{r})\,[\,u(t)\alpha(\mathbf{r})+v(t)\beta(\mathbf{r})\,]\end{aligned} \tag{2.37}$$

with

$$u(t) = \mathrm{Re}\;\sigma_{ge}(t)\,e^{-i[\omega_L t+\Phi(\mathbf{r})]} \tag{2.38.a}$$

$$v(t) = \mathrm{Im}\;\sigma_{ge}(t)\,e^{-i[\omega_L t+\Phi(\mathbf{r})]} \tag{2.38.b}$$

In all these equations, $\mathbf{r}$ means $\mathbf{r}_0+\mathbf{v}_0 t$.

2.5. Optical Bloch Equations (O.B.E.)

The force operator (2.34) depends on the internal atomic operator $\mid e\rangle\langle g \mid$, and the mean force (2.37) depends on the average value of $\mid e\rangle\langle g \mid$, i.e. on the off diagonal element σ_{ge} of the internal atomic density matrix σ. We now briefly explain how it is possible to derive the equations of motion of σ, which are called the optical Bloch equations (for more details, see Ref. 9 and Ref. 2, Complement A_V).

We start from the equations of motion of the four operators

$$\Pi_{ab} = \mid a\rangle\langle b \mid \tag{2.39}$$

with $a, b = e$ or g, which can be written

$$i\hbar\,\dot{\Pi}_{ab} = [\,\Pi_{ab},\, H\,] = [\,\Pi_{ab},\, H_A^{\mathrm{int}} + V_{AL} + V_{AV}] \tag{2.40}$$

since Π_{ab} commutes with $H_A^{\mathrm{ext}} = \mathbf{P}^2/2M$ and H_V. As in section 2.2 above, we replace the field operator $a_j(t)$ appearing in the mode expansion of V_{AV} [see (2.5) and (2.6)] by the solution (2.12) of the Heisenberg equation for $a_j(t)$. One can then show that, if the normal order has been chosen in (2.40), the contribution of the source field, $a_j^{\mathrm{source}}(t)$, to the rate of variation of Π_{ab} reduces to damping terms, proportional to the spontaneous emission rate Γ. On the other hand, the contribution of

the free field appears as a Langevin force with zero average value. The structure of the equation of motion of Π_{ab} is thus the following

$$\dot{\Pi}_{ab} = -\frac{i}{\hbar}[\,\Pi_{ab},\, H_A^{\text{int}} + V_{AL}\,] + \text{Damping terms} + \text{Langevin force} \tag{2.41}$$

Taking the average value of (2.41) and using the fact that the Langevin force has a zero average value, we get

$$\dot{\sigma}_{ba} = -\frac{i}{\hbar}\,\langle\, b \,|\, [\, H_A^{int} + V_{AL}\,, \sigma\,] \,|\, a\,\rangle + \left(\frac{\mathrm{d}}{\mathrm{d}t}\sigma_{ba}\right)_{\text{sp}} \tag{2.42}$$

where the damping terms (due to spontaneous emission) have the following form

$$\left(\frac{\mathrm{d}}{\mathrm{d}t}\sigma_{ee}\right)_{\text{sp}} = -\,\Gamma\,\sigma_{ee} \tag{2.43.a}$$

$$\left(\frac{\mathrm{d}}{\mathrm{d}t}\sigma_{gg}\right)_{\text{sp}} = +\,\Gamma\,\sigma_{ee} \tag{2.43.b}$$

$$\left(\frac{\mathrm{d}}{\mathrm{d}t}\sigma_{eg}\right)_{\text{sp}} = -\,\frac{\Gamma}{2}\,\sigma_{eg} \tag{2.43.c}$$

$$\left(\frac{\mathrm{d}}{\mathrm{d}t}\sigma_{ge}\right)_{\text{sp}} = -\,\frac{\Gamma}{2}\,\sigma_{ge} \tag{2.43.d}$$

Equations (2.43.a) and (2.43.b) describe the departure of the atom from e by spontaneous emission and its transfer to g with a rate Γ. Equations (2.43.c) and (2.43.d) describe the damping of optical coherences with a rate $\Gamma/2$. (Strictly speaking, there are also terms describing a radiative shift of the evolution frequency of the optical coherences, but this shift is supposed to be reincluded in the atomic frequency ω_A.)

From (2.42) and (2.43), it is clear that

$$\frac{\mathrm{d}}{\mathrm{d}t}(\sigma_{ee} + \sigma_{gg}) = 0 \tag{2.44}$$

since the trace of a commutator is zero. It follows that $\sigma_{ee} + \sigma_{gg}$ is constant and equal to 1 and that the four matrix elements σ_{ab} are not

independent. We introduce the so-called Bloch vector with three independent components u, v, w, where u and v are given by (2.38.a) and (2.38.b) and where

$$w(t) = \frac{1}{2}\left[\sigma_{ee}(t) - \sigma_{gg}(t)\right] \tag{2.45}$$

Note that $u(t)$ and $v(t)$ depend on t, not only through $\sigma_{ge}(t)$ and $\sigma_{eg}(t)$, but also through $\Phi(\mathbf{r}) = \Phi(\mathbf{r}_0 + \mathbf{v}_0 t)$.

From (2.38), (2.45), (2.42) and (2.43), we get, using (2.2) and (2.10), the following equations of motion of u, v, w, written in matrix form

$$\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} -\Gamma/2 & \delta + \dot{\Phi} & 0 \\ -(\delta + \dot{\Phi}) & -\Gamma/2 & -\Omega_1 \\ 0 & \Omega_1 & -\Gamma \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\Gamma/2 \end{pmatrix} \tag{2.46}$$

In these equations,

$$\dot{\Phi} = \mathbf{v}_0 \cdot \nabla\Phi = \mathbf{v}_0 \cdot \boldsymbol{\beta} \tag{2.47}$$

and Ω_1 and Φ are evaluated in $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}_0 t$.

Equations (2.46) look like the usual Bloch equations of NMR. The components u, v, w of the Bloch vector can be considered as the components S_x, S_y, S_z of a fictitious spin 1/2 submitted (in the rotating frame) to two static fields, one along $0z$, proportional to $-(\delta + \dot{\Phi})$, one along $0x$, proportional to Ω_1.

3. Mean radiative force for a two-level atom initially at rest

The general results of Chapter 2 are applied here to the particular case of an atom initially at rest , in a point which we take as the origin of coordinates

$$\mathbf{r}_0 = \mathbf{0} \tag{3.1.a}$$

$$\mathbf{v}_0 = \mathbf{0} \tag{3.1.b}$$

The origin of time can always be chosen in such a way that the phase $\Phi(\mathbf{0})$ of the laser field in $\mathbf{r} = \mathbf{0}$ is zero. The laser electric field (1.2) at the position of the atom can then be written

$$\mathbf{E}_L(\mathbf{0}, t) = \epsilon(\mathbf{0})\,\mathcal{E}(\mathbf{0})\cos\omega_L t = \boldsymbol{\mathcal{E}}_0 \cos\omega_L t \tag{3.2}$$

where $\mathcal{E}_0 = \epsilon(0)\,\mathcal{E}(0)$. We use in the following the simpler notation

$$\Omega_1(0) = \Omega_1 \tag{3.3.a}$$

$$\alpha(0) = \left.\frac{\nabla\Omega_1}{\Omega_1}\right|_{\mathbf{r}=\mathbf{0}} = \boldsymbol{\alpha} \tag{3.3.b}$$

$$\beta(0) = \nabla\Phi\,|_{\mathbf{r}=\mathbf{0}} = \boldsymbol{\beta} \tag{3.3.c}$$

Note finally that, as a consequence of (3.1.b)

$$\dot{\Phi} = \mathbf{v}_0 \cdot \nabla\Phi\,\big|_{\mathbf{r}=\mathbf{0}} = 0 \tag{3.4}$$

3.1. Steady-state solution of optical Bloch equations

Using (3.4) and the fact that the Rabi frequency (3.3.a) is time independent (since the atom is at rest in $\mathbf{r} = \mathbf{0}$), we see that the optical Bloch equations given in (2.46) are here a set of coupled linear differential equations with time independent coefficients. They thus admit a steady-state solution which is easily found to be (see Ref. 2, Chap. V)

$$u_{st} = \frac{\delta}{\Omega_1}\,\frac{s}{1+s} \tag{3.5.a}$$

$$v_{st} = \frac{\Gamma}{2\Omega_1}\,\frac{s}{1+s} \tag{3.5.b}$$

$$w_{st} = -\frac{1}{2(1+s)} \tag{3.5.c}$$

where

$$s = \frac{\Omega_1^2/2}{\delta^2 + (\Gamma^2/4)} \tag{3.6}$$

is called the saturation parameter.

We will also need in the following the steady-state value σ_{ee}^{st} of the population of the upper state, which can be deduced from (2.45), (3.5.c) and the relation $\sigma_{ee}^{st} + \sigma_{gg}^{st} = 1$

$$\sigma_{ee}^{st} = \frac{1}{2} + w_{st} = \frac{1}{2}\,\frac{s}{1+s} \tag{3.7}$$

It clearly appears on (3.7) that σ_{ee}^{st} tends to 1/2 for high saturation parameters ($s \gg 1$).

3.2. Reactive response and dissipative response

Equation (2.37) shows that the mean force $\mathcal{F}$ is the sum of two contributions respectively proportional to u and v. In order to interpret physically these two contributions in steady-state, we first show in this section that u_{st} and v_{st} describe respectively the reactive response and the dissipative response of the atom to the laser excitation.

We take the steady-state average value of the dipole moment operator $\mathbf{d}$ given in (2.8). This gives, using (2.39)

$$\begin{aligned} \langle \mathbf{d} \rangle_{st} &= d\,\epsilon_z (\Pi_{eg} + \Pi_{ge})_{st} \\ &= d\,\epsilon_z (\sigma_{ge}^{st} + \sigma_{eg}^{st}) = 2d\,\epsilon_z \text{ Re } \sigma_{ge}^{st} \end{aligned} \tag{3.8}$$

On the other hand, the definition (2.38) of u and v and the fact that $\Phi(0) = 0$ lead to

$$u_{st} + i\,v_{st} = \sigma_{ge}^{st}\, e^{-i\omega_L t}\, e^{-i\Phi(0)} = \sigma_{ge}^{st}\, e^{-i\omega_L t} \tag{3.9}$$

so that

$$\begin{aligned} \text{Re } \sigma_{ge}^{st} &= \text{Re } \left\{ [u_{st} + i\,v_{st}]\, e^{+i\omega_L t} \right\} \\ &= u_{st} \cos\omega_L t - v_{st} \sin\omega_L t \end{aligned} \tag{3.10}$$

From (3.8) and (3.10) it follows that

$$\langle \mathbf{d} \rangle_{st} = 2d\,\epsilon_z [\, u_{st} \cos\omega_L t - v_{st} \sin\omega_L t \,] \tag{3.11}$$

Comparing (3.11) with the expression (3.2) of the laser electric field at the position of the atom, we conclude that u_{st} and v_{st} are proportional to the components of the mean dipole moment respectively in phase and in quadrature with the driving laser field. They thus describe the reactive response and the dissipative response of the atom to the laser excitation.

The previous result suggests that, in steady-state, the mean energy absorbed per unit time by the atom, and consequently the mean number of photons absorbed per unit time $\langle dN/dt \rangle_{st}$, are related to the dissipative response v_{st}. For the following discussion, it will be useful to establish the equation relating $\langle dN/dt \rangle_{st}$ to v_{st}.

The work dW done during dt by the laser electric field (3.2) acting upon the charge q of the atomic electron is

$$dW = q \cos\omega_L t\ \mathcal{E}_0 \cdot d\mathbf{r} \tag{3.12}$$

where dr is the displacement of the charge during dt. Using $\dot{\mathbf{d}} = q\dot{\mathbf{r}}$, we conclude that the mean energy absorbed by per unit time in steady-state is given by

$$\left\langle \frac{\mathrm{d}W}{\mathrm{d}t} \right\rangle_{st} = \cos\omega_L t \;\; \mathcal{E}_0 \cdot \langle \dot{\mathbf{d}} \rangle_{st} \tag{3.13}$$

If we now use the expression (3.11) of $\langle \mathbf{d} \rangle_{st}$, we get

$$\left\langle \frac{\mathrm{d}W}{\mathrm{d}t} \right\rangle_{st} = -2d\,\epsilon_z \cdot \mathcal{E}_0\,\omega_L [\,v_{st}\cos^2\omega_L t + u_{st}\sin\omega_L t\cos\omega_L t\,] \tag{3.14}$$

Averaging over one optical period gives

$$\left\langle \frac{\mathrm{d}W}{\mathrm{d}t} \right\rangle_{st} = \hbar\Omega_1\,\omega_L\,v_{st} \tag{3.15}$$

where we have used the definition $\hbar\Omega_1 = -d\,\epsilon_z \cdot \mathcal{E}_0$ of the Rabi frequency. Since each absorbed photon provides an energy $\hbar\omega_L$, the mean number of photons absorbed per unit time in steady-state is given by

$$\left\langle \frac{\mathrm{d}N}{\mathrm{d}t} \right\rangle_{st} = \frac{1}{\hbar\omega_L} \left\langle \frac{\mathrm{d}W}{\mathrm{d}t} \right\rangle_{st} = \Omega_1\,v_{st} \tag{3.16}$$

or equivalently

$$\left\langle \frac{\mathrm{d}N}{\mathrm{d}t} \right\rangle_{st} = \Gamma\,\sigma_{ee}^{st} \tag{3.17}$$

since it follows from (3.5.b) and (3.7) that

$$\Omega_1 v_{st} = \Gamma\sigma_{ee}^{st} \tag{3.18}$$

Equation (3.17) has a clear physical meaning. It expresses that, in steady-state, the mean number of photons absorbed per unit time (left-hand side) is equal to the mean number of photons spontaneously emitted per unit time (right-hand side).

3.3. Dissipative force - Radiation pressure

We will call dissipative force the component of the mean force (2.37) which, in steady-state, is proportional to v_{st}

$$\mathcal{F}_{\text{dissip}} = -\hbar\Omega_1\,v_{st}\,\beta \tag{3.19}$$

The physical meaning of $\mathcal{F}_{\text{dissip}}$ is particularly clear when the laser wave is a plane wave with a wave vector $\mathbf{k}_L$

$$\mathbf{E}_L(\mathbf{r},t) = \mathcal{E}_\circ \cos(\omega_L t - \mathbf{k}_L \cdot \mathbf{r}) \tag{3.20}$$

It follows from (3.20) that the phase of the field is then

$$\Phi(\mathbf{r}) = -\mathbf{k}_L \cdot \mathbf{r} \tag{3.21}$$

so that

$$\beta = -\nabla\Phi|_{\mathbf{r}=\mathbf{0}} = -\mathbf{k}_L \tag{3.22}$$

Inserting (3.22) into (3.19) gives in this case, using (3.16)

$$\mathcal{F}_{\text{dissip}} = \hbar\Omega_1 \, \mathbf{k}_L \, v_{st} = \hbar\mathbf{k}_L \left\langle \frac{\mathrm{d}N}{\mathrm{d}t} \right\rangle_{st} \tag{3.23}$$

The interpretation of (3.23) is straightforward. During the time interval $\mathrm{d}t$, the atom absorbs $\mathrm{d}N$ photons and gets a momentum $\mathrm{d}\mathbf{P} = \mathrm{d}N\hbar\mathbf{k}_L$ corresponding to a steady-state mean force

$$\left\langle \frac{\mathrm{d}\mathbf{P}}{\mathrm{d}t} \right\rangle_{st} = \hbar\mathbf{k}_L \left\langle \frac{\mathrm{d}N}{\mathrm{d}t} \right\rangle_{st} \tag{3.24}$$

In the previous argument, we have not considered the momentum associated with spontaneously emitted photons. The reason is that spontaneous emission occurs with equal probabilities in two opposite directions so that the loss of momentum due to the reemission process is zero on the average. The dissipative force is also called radiation pressure, or scattering force, since it originates from absorption-spontaneous emission cycles.

If one uses the expression (3.5.b) of v_{st} and the definition (3.6) of the saturation parameter s, one can write (3.23) in an equivalent form

$$\mathcal{F}_{\text{dissip}} = \hbar\mathbf{k}_L \frac{\Gamma}{2} \frac{\Omega_1^2/2}{\delta^2 + (\Gamma^2/4) + (\Omega_1^2/2)} \tag{3.25}$$

which displays more clearly the dependence of $\mathcal{F}_{\text{dissip}}$ on the various parameters. Plotted as a function of the detuning $\delta = \omega_L - \omega_A$, $\mathcal{F}_{\text{dissip}}$ varies as a Lorentz absorption curve centered about $\delta = 0$, as expected for a dissipative process. Let us consider now the variations of $\mathcal{F}_{\text{dissip}}$

with the laser intensity I_L, which is proportional to Ω_1^2. At low intensity (more precisely for $s \ll 1$), one finds that $\mathcal{F}_{\text{dissip}}$ is proportional to I_L. At high intensity (more precisely for $s \gg 1$), $\mathcal{F}_{\text{dissip}}$ tends to a maximum value given by $\hbar \mathbf{k}_L \Gamma/2$, corresponding to a maximum acceleration

$$\mathbf{a}_{\max} = \frac{\hbar \mathbf{k}_L}{M} \frac{\Gamma}{2} \tag{3.26}$$

It is interesting to discuss some orders of magnitude. In (3.26), $\hbar \mathbf{k}_L/M$ is the recoil velocity $\mathbf{v}_{\text{rec}}$ associated with the absorption or the emission of a single photon. Such velocities are usually quite small, for example on the order of 3cm/s for Sodium or 3mm/s for Cesium. But the number of fluorescence cycles per second can reach (for $s \gg 1$) values equal to $\Gamma/2$ which can be quite high since Γ^{-1} is on the order of a few 10^{-9} s. For example, for Sodium, $\Gamma^{-1} = 16\ 10^{-9} s$, so that $a_{\max}$ is on the order of $10^6\ m/s^2$, i.e. on the order of $10^5\ g$, where g is the acceleration due to gravity. This explains how it is possible to stop an atomic beam with resonant radiation pressure in a small distance, on the order of one meter (see W.D.Phillips's lectures).

3.4. Reactive force - Dipole force

We will call $\mathcal{F}_{\text{react}}$ the component of the mean force (2.37) which, in steady-state, is proportional to u_{st}

$$\mathcal{F}_{\text{react}} = -\hbar \Omega_1\, u_{st}\, \boldsymbol{\alpha} \tag{3.27}$$

In a laser plane wave, $\epsilon\, \mathcal{E} \cos(\omega_L t - \mathbf{k}_L \cdot \mathbf{r})$, the amplitude $\mathcal{E}$ and the polarization ϵ of the laser field are independent of $\mathbf{r}$, so that $\boldsymbol{\nabla} \Omega_1$, and consequently $\boldsymbol{\alpha}$, vanish [see the definitions (2.9) of Ω_1 and (2.36.a) of $\boldsymbol{\alpha}$]. It follows that $\mathcal{F}_{\text{react}} = 0$ in a plane wave. The reactive force can appear only if the laser wave is a linear superposition of several plane waves. On the other hand, $\mathcal{F}_{\text{react}}$ cannot involve a net absorption of energy by the atom since it is associated with the reactive response of the atom. These two properties suggest that $\mathcal{F}_{\text{react}}$ is associated with a *redistribution* of photons between the various plane waves forming the laser wave. Photons are removed from one plane wave by absorption processes and transferred into another plane wave by stimulated emission processes. During such a redistribution, the energy of the field does not change since all plane waves have the same frequency ω_L. There is no net absorption of energy by the atom. But, since the momenta of

the photons associated with the various plane waves are not the same, such a redistribution changes the total momentum of the field, and consequently the momentum of the atom.

In order to make such an argument more explicit, let us consider the simple case where the laser wave is formed by two plane waves with wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ (see also Ref. 2, Chapter V, Subsection C.2.d). Fig. 2 represents, in the complex plane, the complex amplitudes E_1 and E_2 of the two fields in a point $\mathbf{r}$ where they are assumed to be in quadrature, so that the two vectors E_1 and E_2 are perpendicular. We will consider only the reactive response u of the atom to the total field E, which has the same phase as E (or the opposite phase, depending of the detuning). Let u_1 and u_2 be the projections of u onto E_1 and E_2. The component u_1 of u in phase with E_1 does not absorb energy on the wave 1. The same argument holds for u_2 and E_2. On the other hand, u_2 is advanced in phase by $\pi/2$ with respect to E_1, whereas u_1 is retarded by $\pi/2$ with respect to E_2. It follows that if E_1 gains energy by interacting with u_2, E_2 loses energy by interacting with u_1. Furthermore, since $\mid E_1 \mid\mid u_2 \mid = \mid E_2 \mid\mid u_1 \mid$, the energy gained by one wave is exactly equal to the energy lost by the other wave. We understand in this way, on the one hand the existence of a redistribution between the two waves, on the other hand the coherent character of such a redistribution which has a sense ($1 \rightarrow 2$ or $2 \rightarrow 1$) depending on the relative phases of the two waves at the point where the atom is located. Note finally that, depending whether u has the same phase as E or the opposite phase, the sense of the redistribution is different. This explains why the reactive force is an odd function of the detuning.

If one uses the expression (3.5.a) of u_{st} and the definition (3.6) of s, one gets for $\mathcal{F}_{\text{react}}$ the following expression

$$\mathcal{F}_{\text{react}} = -\frac{\hbar\delta}{4} \frac{\nabla\Omega_1^2}{\delta^2 + (\Gamma^2/4) + (\Omega_1^2/2)} \tag{3.28}$$

The reactive force varies with $\delta = \omega_L - \omega_A$ as a Lorentz dispersion curve, as expected for a reactive process. For $\delta < 0$ ($\omega_L < \omega_A$), the reactive force pushes the atom towards the regions of higher intensity since it has the same sign as $\nabla\Omega_1^2$. The opposite result holds for $\delta > 0$ ($\omega_L > \omega_A$). For each value of Ω_1^2, i.e. of the intensity I_L, the value of δ which optimizes $\mathcal{F}_{\text{react}}$ is on the order of Ω_1, the corresponding maximal value of $\mathcal{F}_{\text{react}}$ being on the order of

$$(\mathcal{F}_{\text{react}})_{\text{max}} \simeq \frac{\hbar\nabla\Omega_1^2}{\Omega_1} \simeq \hbar\nabla\Omega_1 \tag{3.29}$$

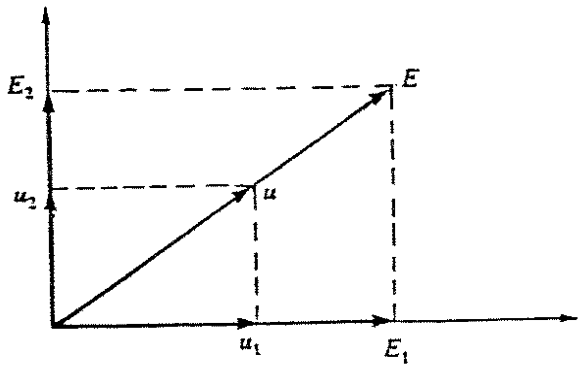

Fig. 2. Representation in the complex plane of a laser field E resulting from the superposition of two fields E_1 and E_2 in quadrature. The vectors u, u_1 and u_2 represent the dipoles in phase respectively with E, E_1 and E_2.

Contrarily to $\mathcal{F}_{\text{dissip}}$ which remains bounded when I_L increases, $\mathcal{F}_{\text{react}}$ increases indefinitely with I_L. Equation (3.29) shows that $\mathcal{F}_{\text{react}}$ can reach values on the order of $\hbar \mathbf{k}_L \Omega_1$ since $\nabla \Omega_1$ can be on the order of $\mathbf{k}_L \Omega_1$, for example in a standing wave. Such a result corresponds to exchanges of momentum $\hbar \mathbf{k}_L$ occurring at a rate Ω_1, as expected for a redistribution process involving absorption-stimulated emission cycles. It has to be compared with the corresponding result for $\mathcal{F}_{\text{dissip}}$ which reaches maximum values on the order of $\hbar \mathbf{k}_L$ times the spontaneous emission rate Γ.

Note finally that the reactive force (3.28) derives from a potential U since one can write

$$\mathcal{F}_{\text{react}} = -\nabla U \tag{3.30}$$

where

$$U(\mathbf{r}) = \frac{\hbar \delta}{2} \operatorname{Ln} \left[1 + \frac{\Omega_1^2(\mathbf{r})/2}{\delta^2 + (\Gamma^2/4)} \right] \tag{3.31}$$

For $\delta < 0$ ($\omega_L < \omega_A$), a region of maximum intensity appears as an attractive potential well for the atom, the maximum depth of such a potential well being on the order of $\mid \hbar \Omega_1^{\max} \mid$.

In the following chapters, we will present other physical pictures for the reactive force, which is also called the dipole force. In Chapter 6, we will interpret the redistribution process at low intensity as resulting